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Seepage Consolidation under Plane Deformation of Elastic Half-Space

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Abstract—The process of seepage consolidation of elastic saturated half-space under the action of an arbitrary normal surface load is investigated in the case of plane deformation. The fluid and skeleton grains are assumed to be incompressible. A new mathematical model of consolidation is proposed with the use of the compatibility equation. Analytic dependences for the sum of effective normal stresses and fluid pressure are found on the basis of this model. The total normal stresses are explicitly expressed in terms of these dependences.

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In the narrow sense, the process of seepage consolidation means deformation of a porous medium, subjected to a compressing surface load, as a result of squeezing the saturated fluid out of the pores. In the wide sense, these are diverse technological actions in chemical productions, building, and extraction of mineral resources which are accompanied with the processes of deformation and flow through a porous medium. The formation and development of the theory of seepage consolidation are related to Terzaghi's studies [1]. In [2] the initial conditions for the plane problem of consolidation were obtained for the uniform load distributed in the form of a strip. In [3] the scheme of calculations of multidimensional problems was considered. In [4–6] the general mathematical model of seepage consolidation and analytic methods of its investigation were proposed. An analysis of the equations of mechanics of saturated elastic media from the position of continuum mechanics was carried out in [7, 8]. Many studies, a part of which is presented in [9, 10], were carried out in the field under consideration. A bibliography was given in [11, 12]. In [13] the plane problem of consolidation under the action of a vertical load was solved for elastic half-space under the assumption that the Terzaghi hypothesis [1, 8] in accordance with which the total stress state in the fluid-rock system is independent of time is fulfilled.

In the present study a new mathematical model of consolidation under the conditions of plane deformation of elastic half-space under the action of an arbitrary vertical load is developed without using the above hypothesis but with utilization of the compatibility equation. The model makes it possible to obtain analytical representations for the fluid pressure and the total normal stresses of the elastic porous skeleton.

The subsequence of solving the problem is following. The equations of the model are written in Section 1. In Section 2 it is shown that the function representing the sum of the effective normal stresses of the skeleton is the solution of the first boundary-value problem in half-plane for the equation of the type of the heat equation. Thereafter, it is established that the auxiliary function introduced as the linear combination of the pressure function and the above-mentioned sum is the solution of the first boundary-value problem in half-plane for the Laplace equation and the pressure can be represented in the form of an integral. At the end of the section, using the equation of motion of the phases, it is shown that one of two total normal stresses can be found as the solution of the first boundary-value problem in half-plane for the Poisson equation. Then, the second stress is also determined.

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